



Rewarding Learning

ADVANCED

General Certificate of Education

Further Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics



AFM11

[AFM11]

Assessment

TIME

2 hours 15 minutes.

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all twelve** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables** booklet is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all twelve questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Show that

$$\sum_{r=1}^{2n} (2r-1)^2 \equiv \frac{2n}{3} (16n^2 - 1) \quad n \geq 1 \quad [6]$$

2 (a) Using the substitution $u = 2x + 1$ show that

$$\int \frac{dx}{\sqrt{4x^2 + 4x + 3}} = \frac{1}{2} \ln \left(2x + 1 + \sqrt{4x^2 + 4x + 3} \right) + c \quad [6]$$

(b) Evaluate the improper integral

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx \quad [4]$$

3 (i) Express

$$f(x) = \frac{2x^2 + x + 7}{(x-1)(x^2 + 4)}$$

in partial fractions.

[7]

(ii) Hence show that

$$\int f(x) dx = 2 \ln(x-1) + \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + c \quad [3]$$

4 (i) Given that

$$I_n = \int (\ln x)^n dx$$

show that for $n \geq 1$

$$I_n = x (\ln x)^n - n I_{n-1} \quad [6]$$

(ii) Evaluate

$$\int_1^2 (\ln x)^2 dx \quad [4]$$

5 Using mathematical induction, prove that

$$\sum_{r=1}^n 3^{r-1} = \frac{3^n - 1}{2} \quad n \geq 1 \quad [6]$$

6 The rate at which a bank's assets grow is modelled by

$$t \frac{dA}{dt} - A = t$$

where A represents the assets of the bank in £ millions and t is the number of years since the setting up of the bank.

(i) Given that the assets of the bank are £300 million after 20 years, find a formula expressing A in terms of t . [8]

(ii) Estimate the assets of the bank 50 years after set up. [2]

(iii) State possible limitations of the model. [2]

7 (a) (i) Given that

$$f(x) = \ln(2x + \sqrt{1 + 4x^2})$$

show that

$$f'(x) = \frac{2}{\sqrt{1 + 4x^2}} \quad [5]$$

(ii) Hence find the Maclaurin expansion of $f(x)$ up to and including the term in x^3 [4]

(b) Using small angle approximations, show that

$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos 6x - 2x \sin 3x}{x^2} \right) = 12 \quad [4]$$

8 (a) Using the exponential definitions of hyperbolic functions, prove that

$$\cosh(x + y) \equiv \cosh x \cosh y + \sinh x \sinh y \quad [6]$$

(b) Solve the equation

$$4 \cosh x + 5 \sinh x = 2$$

leaving the answer in logarithmic form. [7]

9 (i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 10y = 20x - 6 \quad [10]$$

(ii) Given that $\frac{dy}{dx} = 6$ and $y = 0$, when $x = 0$, find y in terms of x . [5]

10 (a) The polar equation of a curve C is

$$r = 3 \sin 2\theta$$

Find a cartesian equation for C.

[6]

(b) Fig. 1 below shows the graph of the finite region bounded by the half-lines

$\theta = 0$ and $\theta = \frac{\pi}{4}$ and the curve with polar equation $r = a(1 + \tan \theta)$

where a is a positive constant.

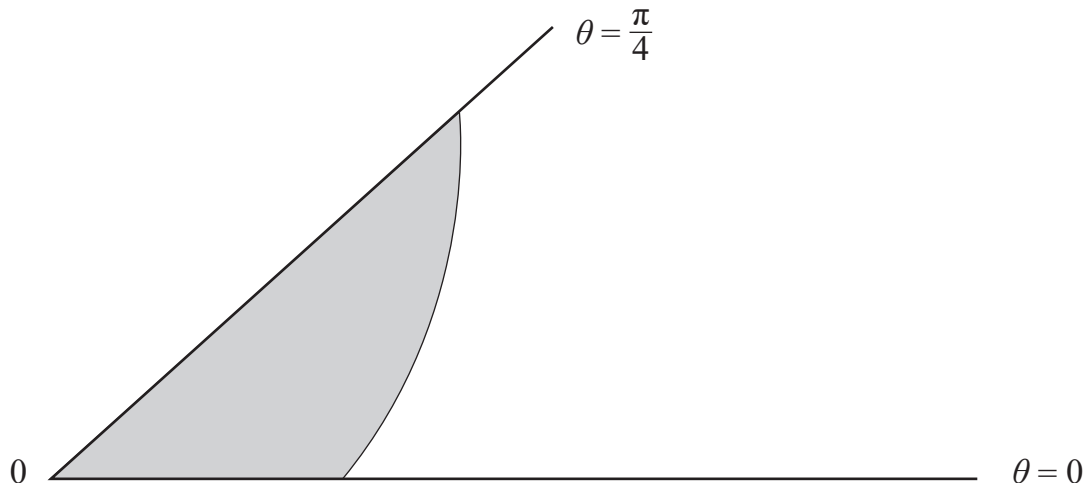


Fig. 1

Find, in terms of a , the area of the shaded region.

[8]

11 (a) Given that n is a positive integer, show that

$$(1 + i)^{4n} - (1 - i)^{4n} = 0 \quad [8]$$

(b) (i) Using De Moivre's theorem, express $\cos 4\theta$ in terms of $\sin \theta$ and $\cos \theta$ [4]

(ii) Using (i) show that

$$\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1 \quad [2]$$

(iii) Hence using $\theta = \frac{\pi}{12}$, show that

$$\cos \frac{\pi}{12} = \frac{1}{2} \sqrt{2 + \sqrt{3}} \quad [7]$$

12 (i) By using exponential definitions, show that

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad -1 < x < 1 \quad [6]$$

(ii) Using (i), or otherwise, show that

$$\frac{d}{dx} \left(\tanh^{-1} (\sin x) \right) = \sec x$$

where $x \neq \frac{(2k+1)\pi}{2}, k \in \mathbb{Z}$ [6]

(iii) Hence, or otherwise, find

$$\int_0^{\frac{\pi}{6}} \sec^2 x \tanh^{-1} (\sin x) dx \quad [8]$$

THIS IS THE END OF THE QUESTION PAPER
